Calculating square roots on a Curta calculator

This method can be performed on either a Type 1 or Type 2 calculator, but the examples are written for Type 1. It is based on a technique described at http://www.hpmuseum.org/root.htm (from "*The Museum of HP Calculators*" website).

Theory

This method uses the odd integer series in which the square of a number n can be computed by the sum of the odd integers from 1 to (2n – 1), i.e.:

$1^2 = 1$
$2^2 = 1 + 3$
$3^2 = 1 + 3 + 5$
$4^2 = 1 + 3 + 5 + 7$
$n^2 = 1 + 3 + ... + (2n - 1)$

To make the following algorithm work well on a calculating machine, we use the same series multiplied by 5:

$5 \times 1^2 = 5 \times (1) = 5$
$5 \times 2^2 = 5 \times (1 + 3) = 5 + 15$
$5 \times 3^2 = 5 \times (1 + 3 + 5) = 5 + 15 + 25$
$5 \times 4^2 = 5 \times (1 + 3 + 5 + 7) = 5 + 15 + 25 + 35$
$5 \times n^2 = 5 \times (1 + 3 + ... + (2n - 1)) = 5 + 15 + ... + (10n - 5)$

This is more efficient for the machine because it turns multiplications by 20 into multiplications by 100, which can be done by shifting the carriage.

Method

(SR = setting register, RR = result register)

1. Set carriage to position 6*, and then multiply the square by 5 (result is in the RR).
2. On the SR, set '50000' (i.e. in digits 5…1).
3. If the 'square × 5' (now in the RR) has an even number of digits to the left of the decimal point (see Example 1 below), rotate the carriage (if necessary) to align the SR '5' with the second most significant digit of the RR. But if it's an odd number of digits to the left of the decimal point (see Example 2 below), align the SR '5' with the third most significant digit of the RR.
4. Subtract (one turn, handle up).
5. If no 'overdraft' (underflow), set '1' on the SR digit to the left of the '5', and subtract once more.
6. If still no underflow, increment the '1' to '2' and subtract once more.

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* If this overflows the most-significant result digit, use a lower carriage position until the multiplicand just correctly occupies the most significant digits of the result register. On the other hand if the square is small and doesn't fill the most significant digits of the RR, put the square in higher digits of the SR so that, when multiplied by 5 they occupy the uppermost RR digits (see Example 2 below).

7. Continue incrementing the left-most digit and subtracting once until an underflow occurs.
8. Handle down and add once to reverse the underflow.
9. Retain the left-most SR digit (call it X), clear the '5' to zero and set '5' on the next SR digit to its right.
10. Rotate the carriage one position clockwise.
11. Repeat steps 4…8, except increment the SR digit to the left of the '5' (i.e. between X and the '5').
12. Retain X and the digit to its right (Y), clear the '5' and set '5' on the next SR digit to its right.
13. Rotate the carriage one more position clockwise, then continue the process described above. Each iteration gives one more answer digit in the SR, and the '5' keeps moving to the right while the carriage rotates clockwise.
14. At some stage after underflowing and adding back the SR value, the result register might show all zeroes (if you're lucky). In this case set the SR's last '5' back to zero, and the other digits to its left represent the square root of the original number. But if the square root can't be expressed in six significant digits (there's a remainder in the RR), then you will need to observe the final underflow RR value and the final remainder after adding back the last SR value, and see which is closest to zero. If the underflowed value was closest to zero (after allowing for the rollover), add '1' to the answer (see Example 2 below).
15. You should be able to determine where the decimal point goes (if applicable) by guessing the approximate square root value (order of magnitude).

Example 1

Find the square root of 191844:

Set the SR's lowest 6 digits to 191844.

Set the carriage to position 6.

Handle down, 5 turns (multiply by 5), RR = 959220.00000.

Now set SR to 50000.

Leave the carriage at position 6 (since the RR has 6 whole digits; the SR '5' aligns with the RR '5').

Handle up (subtraction), 1 turn.

Set SR to 150000, handle up, 1 turn.

Set SR to 250000, handle up, 1 turn.

Set SR to 350000, handle up, 1 turn.

Set SR to 450000, handle up, 1 turn.

Underflow has occurred, so handle down for 1 turn to add back last term (RR = 159220.00000).

Leave the SR '4', clear the '5', and set next digit to right to '5' (so now SR = 405000).

Rotate carriage to position 5.

Repeat subtractions as above, going through SR values of

405000
415000
425000
435000

at which value an underflow occurs. Handle down, 1 turn to add back last term (RR = 34720.00000).

Retain the SR ‘43’, clear the ‘5’ and set next digit to ‘5’ (so SR = 430500).

Rotate the carriage and repeat the above subtraction process, going through SR values of

430500
431500
|
438500

at which value an underflow occurs. Handle down, 1 turn to add back last term (RR = 0).

As the RR is zero, you’ve finished. Clear the SR last ‘5’, and read the answer from the SR (= 438).

Example 2

Find the square root of 78.9:

Set the SR digits 5…3 to 789.

Set the carriage to position 6.

Handle down, 5 turns (multiply by 5), RR = 394.50000000.

Now set SR to 50000.

Set the carriage to position 5 (since the RR has 3 whole digits; the SR ‘5’ aligns with RR ‘4’).

Handle up (subtraction), 1 turn.

Set SR to 150000, handle up, 1 turn.

Set SR to 250000, handle up, 1 turn.

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Set SR to 850000, handle up, 1 turn.

Underflow has occurred, so handle down for 1 turn to add back last term (RR = 74.50000000).

Leave the SR ‘8’, clear the ‘5’, and set next digit to right to ‘5’ (so now SR = 805000).

Rotate carriage to position 4.

Repeat subtractions as above, going through SR values of

805000
815000
|
885000

at which value an underflow occurs. Handle down, 1 turn to add back last term (RR = 7.30000000).

Retain the SR ‘88’, clear the ‘5’ and set next digit to ‘5’ (so SR = 880500).

carriage position 3, repeat the above subtraction process, going through SR values of

880500
881500
|
888500

at which value an underflow occurs. Handle down, 1 turn to add back last term (RR = 0.22800000).

Retain the SR ‘888’, clear the ‘5’ and set next digit to ‘5’ (so SR = 888050).

carriage position 2, repeat the above subtraction process, going through SR values of

888050
888150
888250

at which value an underflow occurs. Handle down, 1 turn to add back last term (RR = 0.05038000).

Retain the SR ‘8882’, clear the ‘5’ and set next digit to ‘5’ (so SR = 888205).

carriage position 1, repeat the above subtraction process, going through SR values of

888205
888215
|
888255

at which value an underflow occurs (RR = 999.99708620 = –0.0029138). Handle down, 1 turn to add back last term (RR = 0.00596875).

As we’ve run out of SR digits, you’ve finished. Clear the SR last ‘5’, and read the answer from the SR (= 88825). But since the RR underflow value was closer to zero than the restored remainder, add ‘1’ to the answer, = 88826.

You know that 9 squared is 81, so the square root of 78.9 will be something less than 9. Hence the decimal point is placed to read 8.8826.

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